Generation of Rainfall Intensity Duration Frequency Curves For Ungauged Sites

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Abstract

The rainfall intensity-duration-frequency (IDF) relationship is commonly required for planning and designing of various water resource projects. Many sets of relationships have been developed and used in several parts of the world. The IDF relationship is a mathematical relationship between the rainfall intensity, the duration and the return period. This relationship is determined through statistical analysis of samples of records at proper meteorological stations. Sinai Peninsula in the North East part of Egypt has a network of daily rainfall recording rain gauges since 1988. A total of six different durations ranging from 5 minutes to 24 hours for return periods of 2, 5, 10, 25, 50, 100 and 200 years were analyzed. The IDF curves and isopluvial maps for the region are developed using the available rainfall data. The main aim of this paper is to construct IDF curves for the region using rainfall frequency analysis techniques. Also, the paper proposes regional IDF formula to estimate rainfall intensity for various return periods and rainfall durations at ungauged sites.

***Key words**:* Rainfall intensity, return period, rainfall duration, isopluvial maps, intensity-duration-frequency relationship (IDF)

1. INTRODUCTION

Rainfall is an important component in the hydrologic cycle. Quantification of rainfall is needed for planning and designing of various water resource projects. Quantification of rainfall is generally done using isopluvial maps and intensity-duration-frequency (IDF) curves (Chow et al., 1988). Hershfield (1961) developed various rainfall contour maps to provide the design rain depths for various return periods and durations. The IDF relationship is a mathematical relationship between the rainfall intensity i, the duration d, and the return period T (the annual frequency of exceedance). The establishment of IDF relationships goes back to the 1930's (Bernard 1932). Since then, different forms of relationships have been constructed for several regions of the world. Also, the regional properties of IDF relationships have been studied in several countries, and in general, maps have been constructed to provide the rainfall intensities or depths for various return periods and durations (IHP-VII, 2008).

IDF curves received considerable attention in engineering hydrology over the past decades. Approaches based on statistical analysis of data were developed, e.g. Bell (1969) and Chen (1983) derived the IDF formulae for the United States. Koutsoyiannis et al. (1998) proposed a new generalizing approach to the formulation of IDF curves using efficient parameterization. Nhat et al. (2006) have established IDF curves for the Monsoon area of Vietnam with two main procedures. The first produced the set of IDF curves at 7 stations by using empirical functions. The second produced a generalized IDF equation for location area. Raiford et al. (2007) have updated the existing IDF curves in the region and obtained these curves at ungauged sites throughout the region using the newly developed rainfall frequency analysis techniques. Finally, the depth-duration-frequency curves and isopluvial maps for the region encompassing South Carolina, North Carolina, and Georgia were developed.

Kim et al. (2008) improved the accuracy of IDF Curves by using long and short duration separation technique. They derived the intensity-duration-frequency (IDF) curve by using cumulative distribution function (CDF) of the interesting site and multi-objective genetic algorithm (MOGA). Application results showed that the developed IDF curve is more accurate than the previously suggested IDF curves, and the duration separation method would be applied automatically without hand calculation. Ben-Zvi (2009) proposed a procedure for basing intensity–duration–frequency (IDF) curves on partial duration series (PDS) which are substantially larger than those commonly used for this purpose.

He concluded that the proposed procedure superior to the current ones where the use of large samples would reduce the sensitivity of predicted intensities to sampling variations.

Bara et al. (2009) applied the simple scaling theory to the intensity-duration-frequency (IDF) characteristics of short duration rainfall. The IDF relationships, which were deduced from daily rainfall, showed acceptable results in comparison with the IDF curves obtained from at-site short-duration rainfall data. Okonkwo and Mbajiorgu (2010) have developed IDF curves for South Eastern Nigeria using two methods, Graphical and Statistical and the results were compared. IDF data developed from the graphical and statistical methods were very close for the lower return periods of 2 to 10 years, but differ for higher return periods of 50 to 100 years, but the difference was not significant at 5% level.

In this paper, IDF curves and isopluvial maps were developed for six different durations of 5, 15, 30, 60, 120 and 1440 minutes) and seven different return periods of 2, 5, 10, 25, 50, 100 and 200 years. Also, the paper proposes the approach for formulation and construction of IDF curves using data from recording station by using empirical equation and determining the equation parameters for the region. The first step in the process is data collection and preparation. Annual Maximum rainfall Series (AMS) is computed for the six selected durations at each station. Next, an appropriate frequency distribution is fitted for AMS at each station for all the selected durations. Spatial analysis is performed to interpolate rainfall values between stations to create the isopluvial maps. Then, point rain stations are used to drive the set of IDF curves at the recording stations by using empirical function. Finally, the regional IDF formula parameters are generated for ungauged sites to estimate rainfall intensity for various return periods and rainfall durations. The main objective of the paper is to construct IDF curves, isopluvial maps and proposes regional IDF formula parameters.

2. DATA COLLECTION AND PREPARATION

Sinai Peninsula in the North East part of Egypt has a network of daily rainfall recording rain stations since 1988 as shown figure (1). There are two types of rainfall stations; recording and digital stations. The advantage of this data is that the rainfall is recorded every minute which facilitate the development of the IDF curves for this area. The errors in the data are removed through quality control and verification analysis. Also, these rainfall measurements are checked with the General Metrological Authority (GMA) measurements. Table (1) gives summary for the information of the recording stations. As shown in table (1), the elevations of the stations are variable from one to another and accordingly the amounts of rainfall vary from location to another. It should be mentioned that the maximum event in table (1) is over the recording period and some of these stations recorded higher values but the details of these event are not available.

The length of the available records and number of analyzed events are listed in table (1). In some situations, when only a few years of data are available (less than 20 to 25 years), an annual exceedence series for each duration can be used. In this case the determined design rainfall depths need to be adjusted for low return periods (less than 10 years) to match those derived from an annual maximum series (Chow et al., 1988). It is generally accepted that data should not be extrapolated more than twice the record length (Raiford et al., 2007).

A Matlab model is prepared to compute the rainfall depths for durations; 5, 15, 30, 60, 120 and 1440 minutes for all available rainfall events of 12 stations. Then, the Annual Maximum rainfall Series (AMS) for various rainfall durations is calculated. Also, there are two sites, as shown in figure (1), are used; one is used for verification (El Shiekh Attia) and the other is used as ungauged site (El Melez).

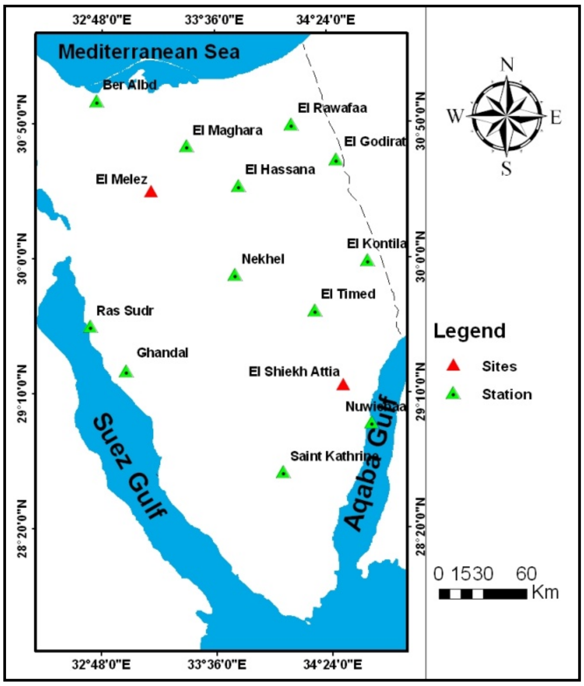


Figure 1: Location of Rain stations

Table 1: Number of recorded years and events

Station Name	Elevation	Observation period	No. of years	No. of Event	Max. Event (mm)	Ave. duration (hr)
El Maghara	405	1990-2010	17	44	27.2	3.3
El Timed	625	1990-2010	8	25	38.6	3.7
El Godirat	390	1990-2010	18	53	20	5.6
El Kontila	530	1990-2010	11	25	9.6	4.4
Ghrandal	69	1992-2010	17	48	19.3	3.3
EL Rawafaa	136	1990-2010	20	61	25	6.6
El Hassana	250	1991-2010	15	26	17.4	5.1
Ber Albd	18	1990-2010	16	53	26.8	6.1
Nekhel	402	1990-2010	6	22	23.6	1.7
Saint Kathrine	1652	1990-2010	17	38	25.02	4.7
Ras Sudr	290	1990-2010	20	51	27.4	4.6
Nuwiebaa	50	1988-2010	9	20	16.7	3.3

3. METHODOLOGY

The main characteristics of a storm are its intensity, duration, total amount and frequency or recurrence interval. The rainfall intensity is an important characteristic of rainfall storm and it is expressed as the rate of rainfall in inches or millimeter per hour. Similarly, rainfall duration is the period of time that rain falls at a particular rate or intensity. For the rainfall storm, the rainfall intensity may vary from high to very low; hence, the duration is how long time rainfall intensity lasts at a particular rate. Frequency is how often a storm of specified intensity and duration may be expected to occur.

$$P = 1/T \quad (1)$$

$$F = 1 - 1/T \quad (2)$$

$$T = 1/(1-F) \quad (3)$$

Where:

P: Probability of exceedence (that an event of specified depth and duration will be exceeded in a year)
T: Return Period (average length of time between events of a given depth and duration)
F: Frequency (how rate or frequent the event is)
For many hydrologic analyses and planning projects, reliable rainfall intensity estimates are required. Rainfall intensity duration frequency relationship IDF comprises the estimates of rainfall intensities of different durations and recurrence intervals. The Methodology for establishment the IDF curves can be summarized in the following three steps:

First step: is to relate the maximum rainfall intensity or depth for each time interval with the corresponding return period from the probability distribution function. There are a number of probability distribution functions that can be used such as; Type I Extreme value (Gumbel), General Extreme value (GEV), Weibull, Normal, Log-normal, Pearson Type III and Log- Pearson Type III distributions.
The second step: is to calculate the rainfall intensities for each duration (from 5 minutes to 1440 minutes) corresponding to different return periods; 2, 10, 25, 50, 100 and 200 years.
The third step: is to use the empirical formula to construct the rainfall IDF curves. The IDF relationship is a mathematical relationship between the rainfall intensity I, the duration d, and the return period T.

The IDF relation is expressed mathematically as follows:

$$I = f(T, d) \tag{4}$$

The rainfall intensity *I* (mm/hr) is a function of the variables *T* which is the return period (years) and *d* which is the rainfall duration (min). The typical IDF relationship for a specific return period is a special case of the generalized formula as given in equation (5). This expression is an empirical formula that encapsulates the experience from several studies (Nhat et al., 2006).

$$I = a/(d^{v}+b)^{e} \tag{5}$$

where *a, b, e* and *v* are non-negative coefficients. If *e* is taken equal unity, equation (5) yields to Kimijima equation that is given below:

$$I = a/(d^{v}+b) \tag{6}$$

4. PROBABILITY DISTRIBUTION

HYFRAN software (HYFRAN manual 1998) is used to fit statistical distributions. It consists of a number of powerful, flexible, user-friendly mathematical tools that can be used for the statistical analysis of extreme events. Also, it includes a large number of probability distributions such as; Type I Extreme value (Gumbel), General Extreme value (GEV), Weibull, Normal, Log-normal, Pearson Type III and Log- Pearson Type III distributions. Also, it can allow the comparison of fitting results with various statistical distributions on the same data set. The comparison of fitting function is based on number of parameters of the fitted probability distribution and the confidence interval. The best distribution is selected based on HYFRAN comparison of fitting function.

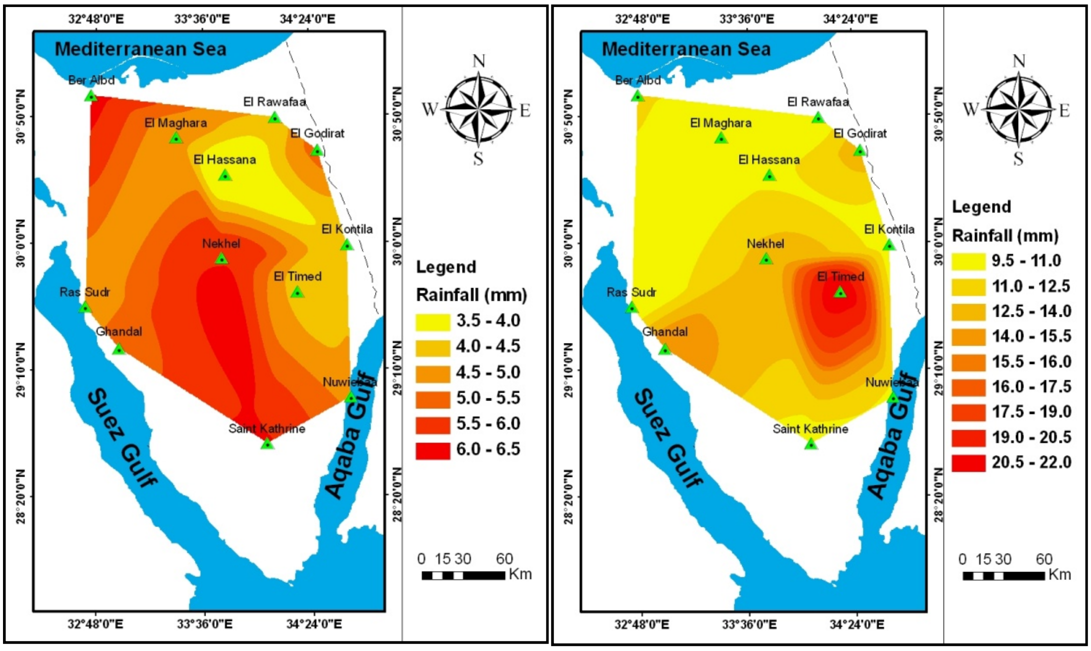
The statistical analysis is carried out using HYFRAN software for both rainfall depth and intensity AMS for six durations at each station. The best fit distribution for each AMS is selected according to the fitting functions. For almost AMS, Log-Normal and Log-Pearson Type III are the best fit distribution. The results of the AMS rainfall depth are used to develop the isopluvial maps. On the other hand, the results of the AMS rainfall intensity are used to establish the IDF curves.

5. DEVELOPING ISOPLUVIAL MAPS

According to the statistical analysis, the best fit distribution is selected for AMS rainfall depth. Accordingly, the maximum rainfall depths for considered durations at different return periods are extracted from the best fit distribution as listed in Table (2). Using ARCGIS, the depth-duration-frequency (DDF) values at different stations are interpolated by the Triangulation with Smoothing method. These values are used to develop isopluvial maps for a given duration over the whole region. Some examples of such maps are shown in Figures (2) and (3) for durations of 30 and 1440 minutes at 5 and 50 years return periods respectively.

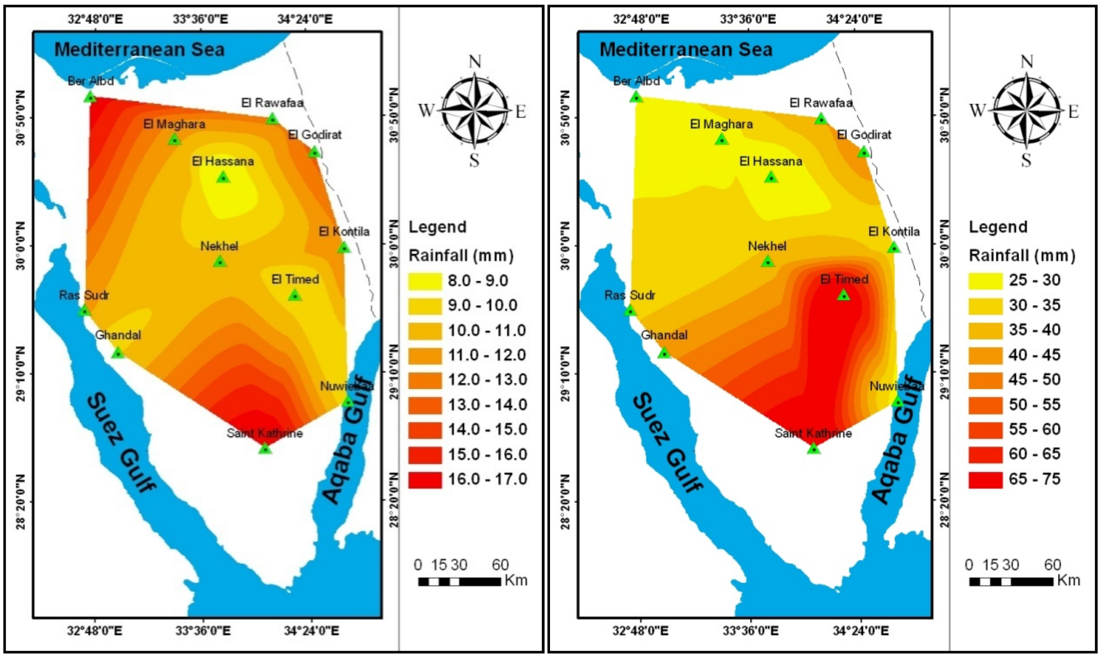
Table 2: Frequency Results of rainfall depth (mm) at El Rawafaa station

Return Period (year)	5-min	10-min	15-min	30-min	60-min	120-min	1440-min
2	1.12	1.75	2.02	2.82	3.66	4.96	6.69
5	1.80	2.70	3.25	4.46	5.67	7.60	12.52
10	2.34	3.43	4.38	5.90	7.46	10.01	18.63
20	2.93	4.21	5.71	7.57	9.53	12.85	26.51
25	3.14	4.47	6.19	8.16	10.26	13.86	29.48
50	3.83	5.34	7.83	10.17	12.75	17.37	40.22
100	4.62	6.28	9.75	12.47	15.61	21.45	53.56
200	5.51	7.32	11.96	15.09	18.87	26.16	69.90



(30-minute, 5-year) (30-minute, 50-year)

Figure 2: Typical Isopluvial Maps (mm) for 30-minute duration



(1440-minute, 5-year) (1440-minute, 50-year)

Figure 3: Typical Isopluvial Maps (mm) for 1440-minute duration

6. ESTABLISHING IDF CURVES

Frequency analysis techniques are used to develop the relationship between the rainfall intensity, storm duration, and return periods from rainfall data. Distribution analysis of rainfall frequency is based on the Log-Normal or Log-Pearson Type III distributions, which is selected by the statistical Software (HYFRAN) as the best distribution for most of the AMS rainfall intensities.

The probability distribution is used to calculate the rainfall intensity at different rainfall durations and return periods to form the historical IDF curves for each station. By using this frequency distribution function, the maximum rainfall intensity for considered durations at 2, 5, 10, 25, 50,100 and 200 years return periods, have been determined as shown in Table (3).

Table 3: Frequency Results of rainfall intensity (mm/hr) at El Godirat station

Return Period (year)	5-min	10-min	15-min	30-min	60-min	120-min	1440-min
2	15.45	12.59	9.62	5.67	3.48	2.28	0.24
5	29.23	23.77	17.86	10.40	6.16	4.14	0.56
10	40.49	32.98	24.71	14.46	8.36	5.62	0.87
20	52.86	43.15	32.31	19.06	10.78	7.23	1.24
25	57.10	46.65	34.94	20.67	11.61	7.78	1.38
50	71.19	58.31	43.70	26.11	14.37	9.59	1.88
100	86.74	71.23	53.45	32.25	17.43	11.56	2.47
200	103.87	85.51	64.26	39.16	20.81	13.71	3.17

The rainfall IDF curves are derived from the point rain stations; only six sets of IDF curves at point are established. The results are shown in Figure (4) at El Godirat station. Then this relationship between the maximum rainfall intensities and the durations for every return period are determined by fitting Kimijima empirical function.

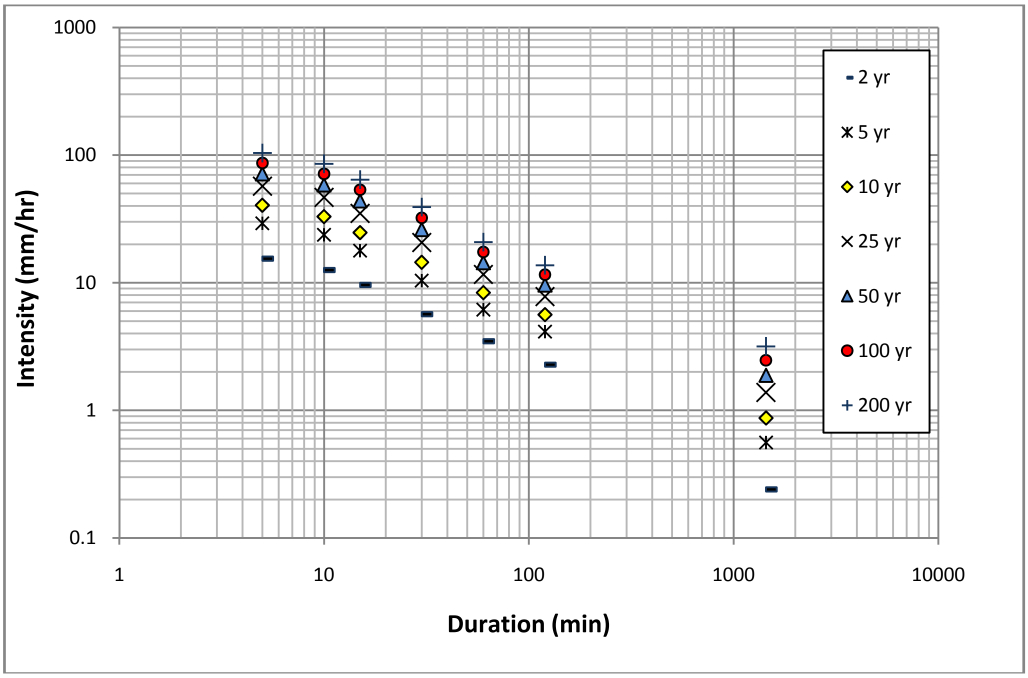


Figure 4: Maximum rainfall intensities for different time intervals and return periods at El Godirat station

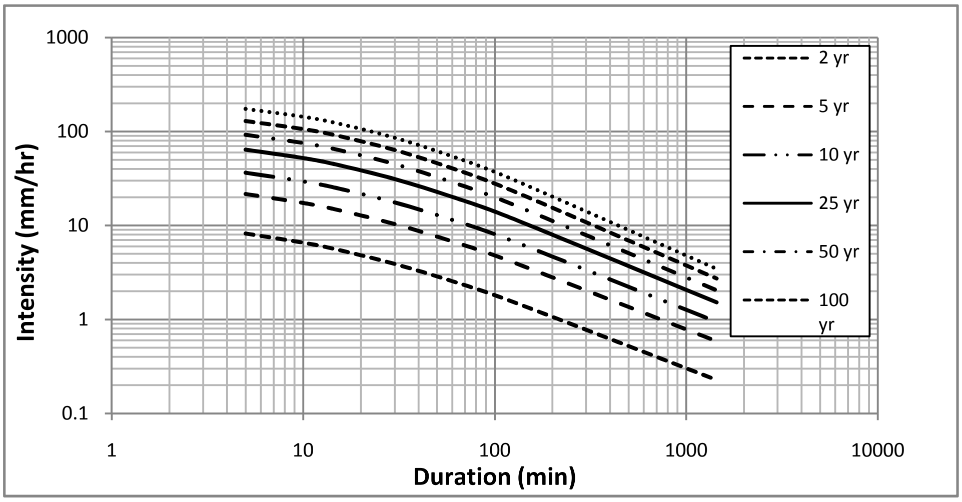
7. EMPIRICAL IDF FORMULAS

The IDF formulas are the empirical equations representing a relationship among maximum rainfall intensity, rainfall duration and frequency. There are several commonly and widely used for hydrology practical applications (Chow et al., 1988). The general form of Kimijima equation is used to describe the rainfall intensity duration relationship. The parameters of this equation are determined based on the minimum of Root Mean Square Error (RMSE) between the IDF relationships produced by the frequency analysis and that simulated by Kimijima equation.

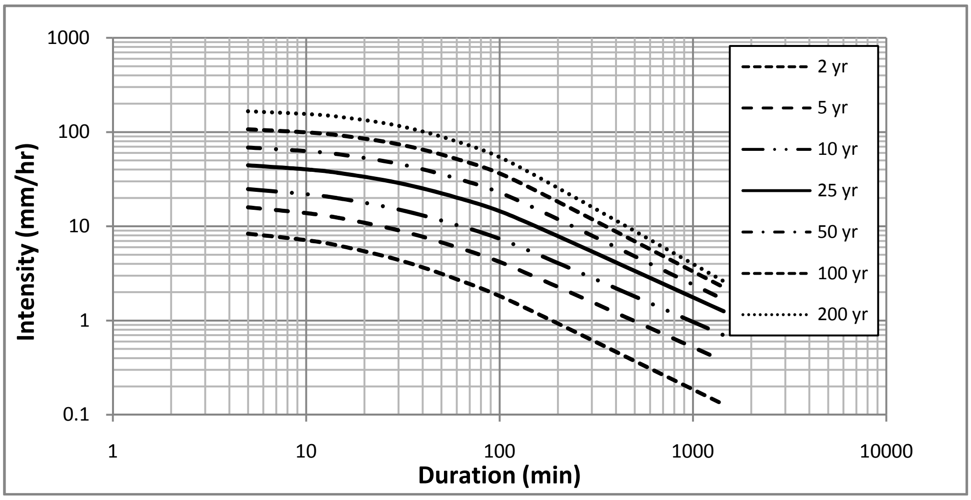
The Kimijima parameters for Ghrandal, El Timid and El Godirat stations are determined and presented in Table (4). Also, the IDF curves for these stations are constructed with the Kimijima equation as shown in Figure (5).

Table 4: Kimijima parameters for Ghrandal, El Timid and El Godirat stations

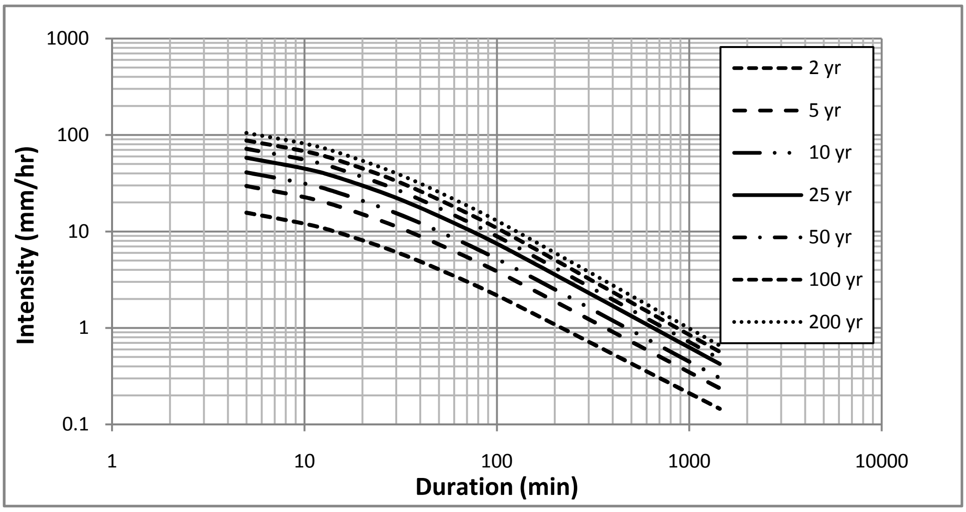
Return Period (yr)	Ghrandal station			El Timid station			El Godirat station		
	a	*v*	*b*	*a*	*v*	*b*	*a*	*v*	*b*
2	96.8	0.83	7.9	290.8	1.06	29.4	303.3	1.05	13.9
5	269.37	0.84	8.6	507.9	0.99	27.1	613.8	1.08	15.0
10	499.2	0.86	9.7	891.9	0.98	31.1	901.9	1.09	16.1
25	1002.6	0.89	11.5	2310.3	1.03	46.9	1364.1	1.11	17.5
50	1585.8	0.91	12.8	5098.4	1.10	68.6	1782.2	1.13	18.6
100	2398.7	0.93	14.1	11382.2	1.17	100.2	2271.6	1.14	19.6
200	3509.6	0.95	15.5	25329.6	1.26	144.9	2830.8	1.15	20.6



(a) Ghrandal station



(b) El Timid

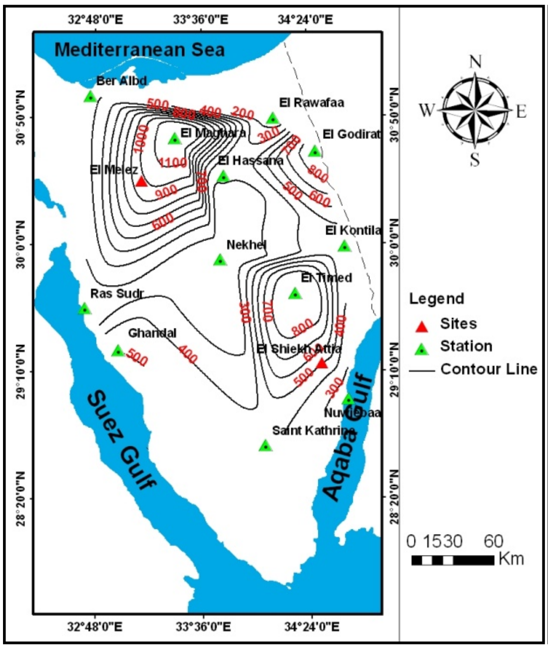


(c) El Godirat

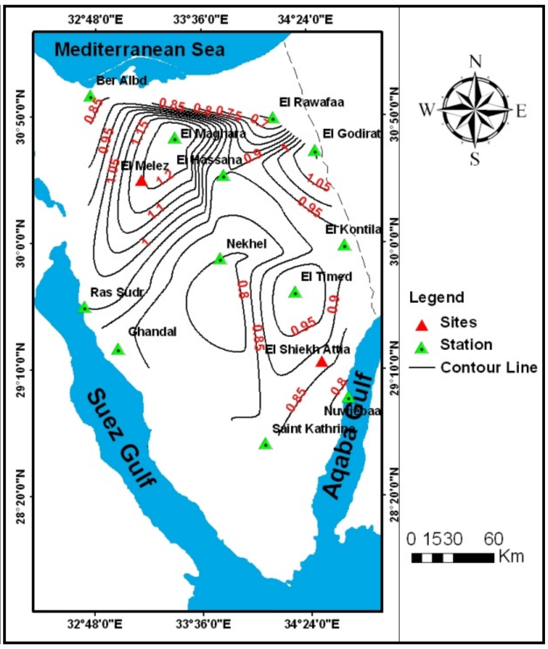
Figure 5: Rainfall Intensity Duration Frequency (IDF) curves at Ghrandal, El Timid and El Godirat stations with Kimijima Equation

8. REGIONALIZED IDF FORMULA

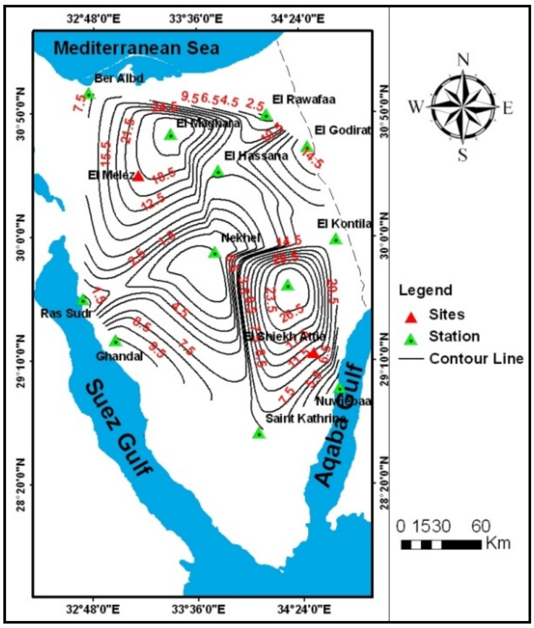
After determining the parameters of IDF formula a, b and e for the same return period, Arc GIS interpolation function is used to get the parameter contour maps. The parameter values at the station points were interpolated by the Triangulation with Smoothing method as shown in Figure (6) and (7). From these maps, it is possible to estimate the parameters set at any point in the region and accordingly the rainfall IDF curves can be constructed by using these parameters.



(a) Parameter *a* contour map

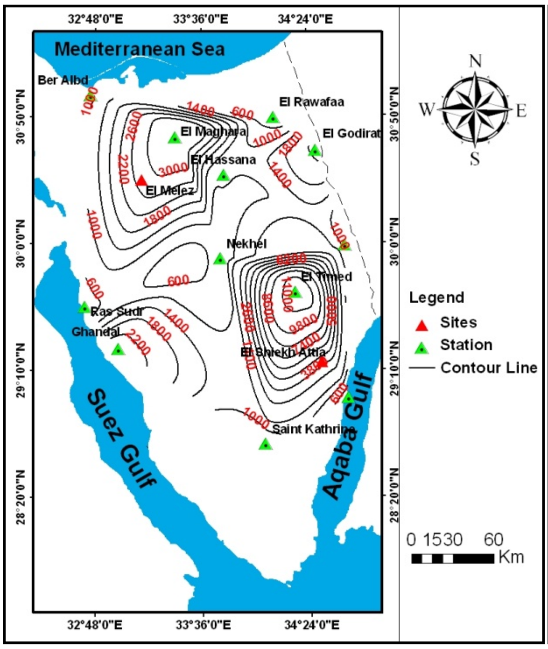


(b) Parameter *v* contour map

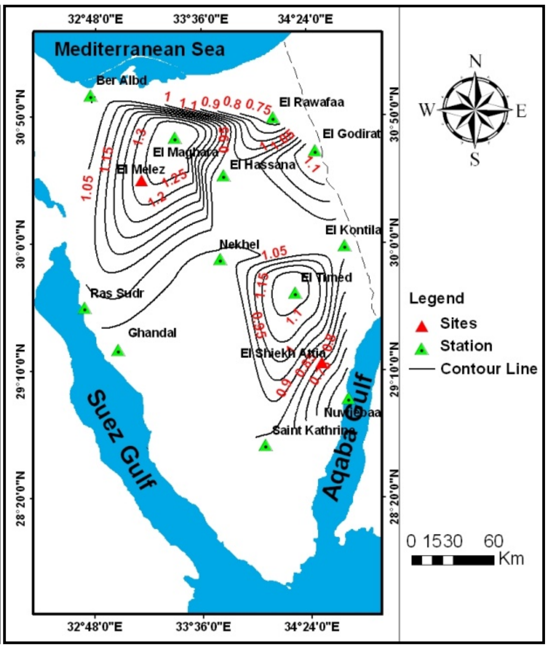


(c) Parameter *b* contour map

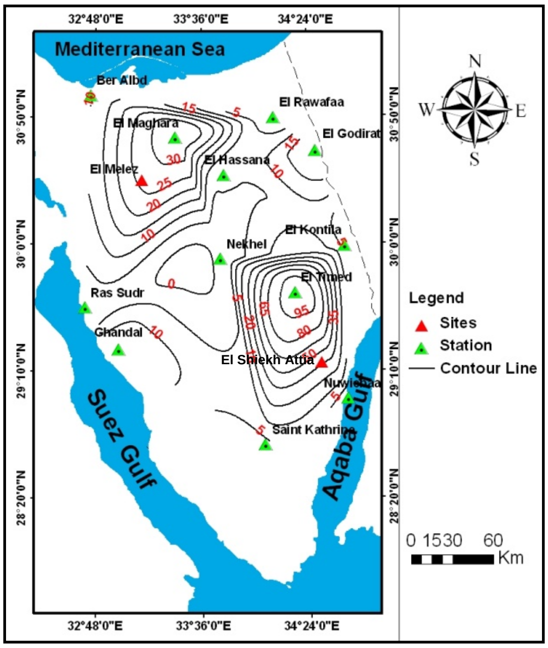
Figure 6: Parameters Contour maps of Kimijima equation with 10-year return period



(a) Parameter *a* contour map



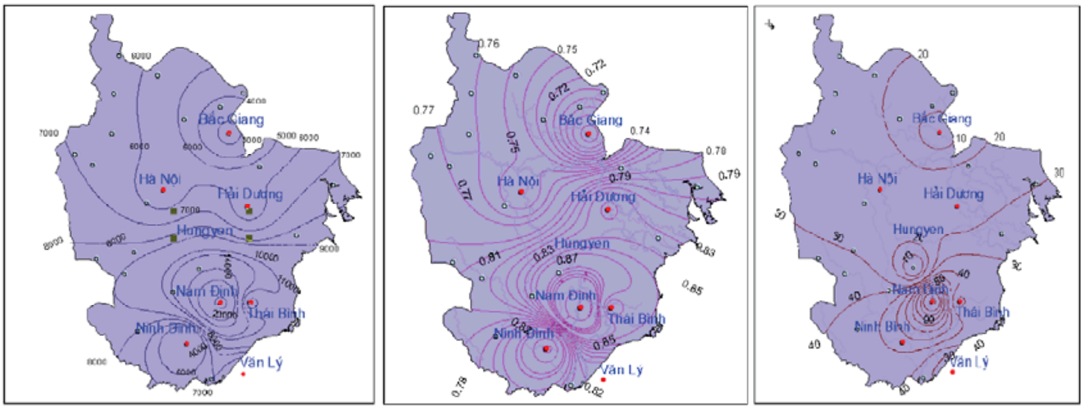
(b) Parameter *v* contour map



(c) Parameter *b* contour map

Figure 7: Parameters Contour maps of Kimijima equation with 100-year return period

According to literature, there is no generation of the IDF curves in the study area and accordingly the parameters contour maps. So the results are compared to available results in Vietnam (Nhat ,2006). Figure (8) shows the parameters of Kimijima equation contour maps for the monsoon area of Vietnam. It can be noticed that the maps are similar in formation of islands and also the values of the Kimijima equation (a, ***v*** and b) are in the same range.



(a) Parameter *a* contour map **(b) Parameter *v* contour map** **(c) Parameter *b* contour map**

Figure 8: Parameters Contour maps of Kimijima parameters for the monsoon area of Vietnam with 100-year return period (Nhat ,2006)

The parameters *a*, *b* and *v* at El Shiekh Attia is determined by using the parameters contour maps set. Then, the IDF curves at these sites are established by using Kimijima equation (equation no. 6) for 10 and 100 years return periods as shown in Figure (9). Also, the available rainfall data for El Shiekh Attia is used to develop the rainfall IDF curves for different durations and return periods. The IDF curves for 10 and 100 years return periods are compared to those established using Kimijima equation as shown in figure (9). The comparison shows good agreement between the corresponding curves especially for low return periods.

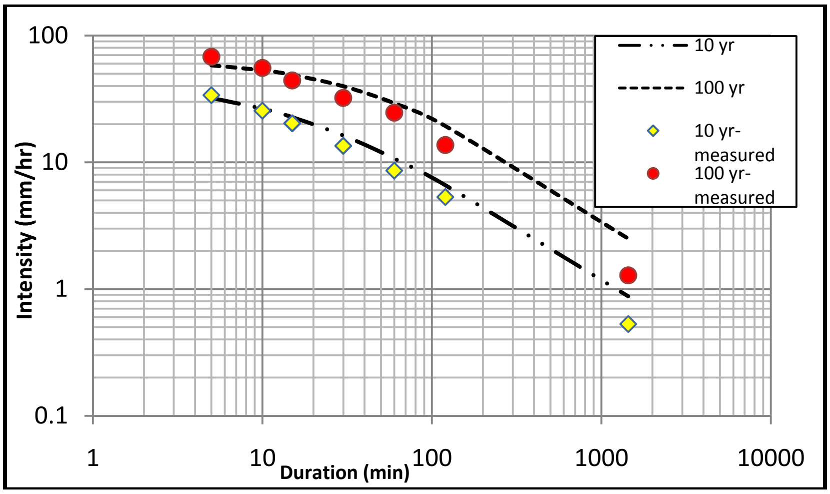


Figure 9: Rainfall IDF curves comparison at El Shiekh Attia

The regional IDF formula parameters are generated for ungauged sites to estimate rainfall intensity for various return periods and rainfall durations. The parameters *a*, *b* and *v* at El Melez is determined by using the parameters contour maps set. Then, the IDF curves at these sites are established by using Kimijima equation (equation no. 6) for 10 and 100 years return periods as shown in Figure (10).

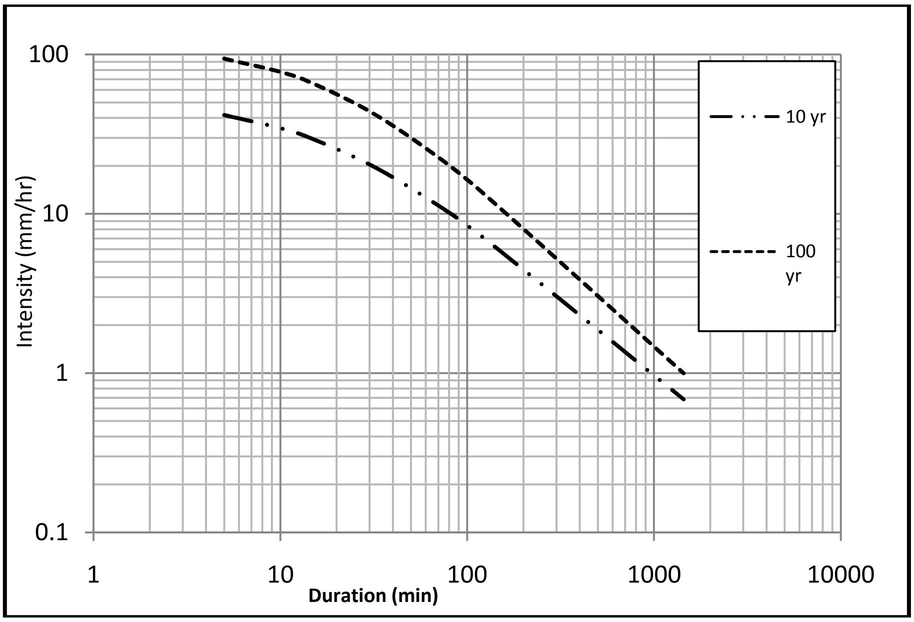


Figure 10: Rainfall IDF curves at El Melez (ungauged site)

9. CONCLUSIONS

The Maximum Annual Precipitation series is obtained at each station for different durations and fitted to one of the statistical distributions; Type I Extreme value (Gumbel), General Extreme value (GEV), Weibull, Normal, Log-normal, Pearson Type III and Log- Pearson Type III distributions. The distribution selected based on fitting comparison criteria of HYFRAN statistical software. Then, this distribution is used to find depth-duration-frequency (DDF) values at 2, 10, 25, 50, 100 and 200 years. These DDF values are spatially interpolated to obtain isopluvial maps for all durations and return periods.

Also, this research has constructed IDF curves using data from recording station by using empirical equation to represent Intensity- Duration-Frequency relationship for Sinai Peninsula in the north east part of Egypt. The three parameters function (Kimijima) has showed acceptable fitting to the rainfall intensity quartiles. The parameters of rainfall intensity-duration-frequency equation are generated to create parameters contour maps. The parameter contour maps are used to estimate these parameters at ungauged sites. Accordingly the IDF curves are constructed for ungauged sites to estimate rainfall intensity for various return periods and rainfall durations.

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